Indeterministic Quantum Gravity and Cosmology

XI. Quantum Measurement

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Abstract

This paper is a sequel to the series of papers [1-10]. We define a quantum measurement as a sequence of binary quantum jumps caused by a macroscopic apparatus. A dynamical theory of measurement is developed, the role of gravity and cosmology being emphasized.

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Measurement began our might.

W. B. Yeats

Introduction

Quantum theory has faced the problem of quantum measurement for more than 70 years. There is voluminous literature devoted to this problem, which cannot be reviewed here. As for a critical part, we restrict our consideration to three aspects of the problem of quantum measurements and quantum jumps in general: energy conservation, decoherence, and nonlocality. We treat a quantum measurement in the light of quantum jump dynamics developed in this series of papers [1-10].

The transformation of a pure state into a mixed one results generally in changing the mean value of the Hamiltonian, so that had a measurement of any observable in any state been possible, the law of conservation of energy would have been violated. Then it would have been possible to construct perpetuum mobile of the third kind—a device increasing energy and entropy. The impossibility of such a device imposes essential restrictions on measurements and quantum jumps in general.

The concept of decoherence claims that it is the latter that causes the transformation of a pure state into a mixed one. But as long as the state amplitude is a vector in a separable Hilbert space, this claim is untenable. Therefore a theory of measurement and quantum jumps in general must not be based on the concept of decoherence.

Quantum entanglement makes possible a situation where a local measurement results in a nonlocal quantum jump. Thus the issue of the relationship between quantum jumps and relativity theory arises. This issue cannot be resolved within the framework of special relativity.

We define a quantum measurement as a sequence of binary quantum jumps caused by a macroscopic apparatus. As for the content of the present paper, the main results are as follows.

In view of the principle of cosmic energy determinacy [1], the law of conservation of energy holds at quantum jumps, so that there is no perpetuum mobile.

Quantum jump dynamics [5] has nothing to do with decoherence, so that the concept of the latter is denounced.

Nonlocality of quantum jumps implies an additional structure of the spacetime manifold, which is absent in special relativity. The structure is this: The hypersurface of a quantum jump is that of a constant value of the cosmic time [1,2,5].

Any quantum jump is binary [5], so that a measurement is, in fact, a sequence of binary jumps.

A macroscopic apparatus causes a quantum jump via the effect of gravitational autolocalization [8,9], due to which the apparatus pointer takes up a definite position.

1   Measurement as quantum jumps

We define a quantum measurement as a sequence of binary quantum jumps caused by a macroscopic apparatus. One binary jump is an elementary measurement.

Let A be an observable measured in a system,

$$A = \sum_i a_i P_i, \tag{1.1}$$

where P_i's are projectors. Let φ, ψ, and Ψ be state vectors relating to the system under measurement, the apparatus, and the composed system respectively. We have:

$$\text{before the measurement} \qquad \Psi_b = \varphi \otimes \psi, \tag{1.2}$$

$$\text{after the measurement} \qquad \Psi_{ai} = \varphi_i \otimes \psi_i \quad \text{with a probability} \quad w_i \tag{1.3}$$

where

$$\varphi_i = \frac{P_i \varphi}{\|P_i \varphi\|}, \qquad w_i = (\varphi, P_i \varphi). \tag{1.4}$$

Statistical operators of the composed system before and after the measurement are

$$\rho_b^{\text{comp}} = P_{\Psi_b}^{\text{comp}} \tag{1.5}$$

and

$$\rho_a^{\text{comp}} = \sum_i w_i P_{\Psi_{ai}}^{\text{comp}} \tag{1.6}$$

respectively, where P_Ψ^{comp} is a projector corresponding to a vector Ψ of the composed system.

2 Against perpetuum mobile of the third kind

The mean value of the energy of the composed system is

$$E = \text{Tr}\{\rho^{\text{comp}} H\}, \tag{2.1}$$

so that the change of the energy is

$$\Delta E \equiv E_a - E_b = \sum_i w_i(\Psi_{ai}, H\Psi_{ai}) - (\Psi_b, H\Psi_b). \tag{2.2}$$

The change of entropy is

$$\Delta\sigma \equiv \sigma_a - \sigma_b = \sigma_a = -\sum_i w_i \ln w_i > 0. \tag{2.3}$$

In the case of measuring an arbitrary observable A we obtain

$$\Delta E \neq 0, \tag{2.4}$$

i.e., the violation of the law of conservation of energy.

According to the first law of thermodynamics

$$\Delta E = Q + W \tag{2.5}$$

where

$$Q = \text{Tr}\{(\Delta\rho^{\text{comp}})H\} \tag{2.6}$$

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is heat and

$$W = \text{Tr}\{\rho^{\text{comp}}(\Delta H)\} \tag{2.7}$$

is work. In our case, eq.(2.2),

$$\Delta E = Q. \tag{2.8}$$

Let $Q > 0$, so that

$$Q > 0, \quad \Delta\sigma > 0. \tag{2.9}$$

Such a self-heating system may be called perpetuum mobile of the third kind.

The requirement that the equality

$$\Delta E = 0 \tag{2.10}$$

must hold imposes essential restrictions on measurements and quantum jumps in general.

In indeterministic quantum gravity and cosmology (IQGC), in view of the principle of cosmic energy determinacy [1], the law of conservation of energy holds at quantum jumps, so that there exists no perpetuum mobile.

3   Against decoherence concept

By the concept of decoherence, the pure state

$$\tilde{\rho}_a^{\text{comp}} = P_{\Psi_a}^{\text{comp}}, \qquad \Psi_a = \sum_i (P_i \varphi) \otimes \psi_i, \tag{3.1}$$

of the composed system is equivalent to the state ρ_a^{comp} given by eq.(1.6),

$$\tilde{\rho}_a^{\text{comp}} = \rho_a^{\text{comp}}, \tag{3.2}$$

eq.(3.2) being fulfilled due to the decoherence condition:

$$(\Psi_{ai'}, O\Psi_{ai}) = 0 \quad \text{for} \quad i' \neq i \tag{3.3}$$

where Ψ_{ai} is given by eq.(1.3) and O is any observable of the composed system.

In IQGC, quantum jump dynamics [5] has nothing to do with the absurd condition given by eq.(3.3), so that the concept of decoherence is denounced.

4   Against overlooking gravity and cosmology in quantum jump dynamics

To conceive of the role of gravity and cosmology in quantum jump dynamics, let us consider a geometric, or coordinate-free description of quantum fields. In the Heisenberg picture, a quantum field is

$$\phi_H \equiv \phi = \phi(p), \qquad p \in M, \tag{4.1}$$

where M is a spacetime,

$$\Psi_H \equiv \Psi \tag{4.2}$$

is a field state vector;

$$\phi^{class}(p) = (\Psi, \phi(p)\Psi) \tag{4.3}$$

is a classical field.

In the Heisenberg picture, the state vector Ψ changes at and only at quantum jumps. To every quantum jump there corresponds a spacelike hypersurface. The hypersurfaces must be mutually disjoint: otherwise Ψ would be not defined. The quantum-jump hypersurfaces are causewise ordered:

$$\mathcal{S}_2 > \mathcal{S}_1, \ \text{ or } \ \mathcal{S}_1 < \mathcal{S}_2. \tag{4.4}$$

Within the limits of special relativity, i.e., if M is the Minkowski spacetime, it is neither theoretically nor experimentally possible to determine what are those hypersurfaces and which is, in the general case, their causelike order. It is special relativity that prevents a complete phenomenological mathematical description of quantum jumps and, by the same token, measurements.

A complete dynamical description of quantum jumps has been given in IQGC. The quantum jumps imply an additional structure of the spacetime manifold, which is absent in special relativity. The hypersurface of a quantum jump is that of a constant value of the cosmic time t, so that

$$\mathcal{S}_2 > \mathcal{S}_1 \Leftrightarrow t_2 > t_1. \tag{4.5}$$

5   Binary jumps and measurement

By eqs.(1.1),(1.4) we have for a measurement:

$$\{P_i : i \in \mathcal{I}\}, \quad \sum_{i\in\mathcal{I}} P_i = I, \tag{5.1}$$

$$w_i = \mathrm{Tr}\{\rho P_i\}, \quad \rho \equiv \rho_b = P_\varphi. \tag{5.2}$$

An elementary measurement is described by a binary quantum jump, for which the equation

$$P^+ + P^- = I \tag{5.3}$$

holds [5]. Now an equation

$$P_i = P^{1s_1\,2s_2...ns_n}, \quad n = n(i), \quad s_k = \pm, \tag{5.4}$$

and relations

$$P^{1s_1...(n-1)s_{n-1}\,ns_n} P^{1s_1...(n-1)s_{n-1}} = P^{1s_1...ns_n}, \tag{5.5}$$

$$P^{1s_1...(n-1)s_{n-1}\,n+} + P^{1s_1...(n-1)s_{n-1}\,n-} = P^{1s_1...(n-1)s_{n-1}} \tag{5.6}$$

hold.

We have for the probability of the i-th result as the sequence given by eq.(5.4):

$$\tilde{w}_i = \mathrm{Tr}\{\rho P^{1s_1}\}\mathrm{Tr}\{\rho^{1s_1} P^{1s_1\,2s_2}\}\cdots\mathrm{Tr}\{\rho^{(n-1)s_{n-1}} P^{1s_1...ns_n}\} \tag{5.7}$$

where

$$\rho^{ks_k} = \frac{P^{1s_1...ks_k}\rho^{(k-1)s_{k-1}} P^{1s_1...ks_k}}{\mathrm{Tr}\{\rho^{(k-1)s_{k-1}} P^{1s_1...ks_k}\}}. \tag{5.8}$$

We find

$$\tilde{w}_i = \mathrm{Tr}\{P^{1s_1...ns_n}\cdots P^{1s_1}\rho P^{1s_1}\cdots P^{1s_1...ns_n}\} = \mathrm{Tr}\{\rho P^{1s_1...ns_n}\} = \mathrm{Tr}\{\rho P_i\} = w_i. \tag{5.9}$$

6 Determinacy of the pointer position

We take a ball as a model of the pointer of a macroscopic apparatus and use the results [8,9] for the ball.

In IQGC the following relations take place:

$$H = H[g, \dot{g}] \tag{6.1}$$

and

$$\ddot{g} = \ddot{g}[g, \dot{g}, \Psi] \tag{6.2}$$

where g is the metric and dot stands for the derivative with respect to the cosmic time t. In the Newtonian approximation, on the other hand,

$$H = H[\Psi], \tag{6.3}$$

so that the approximation is inapplicable at the jump time t_{jump}.

Let at $t = t_{\text{jump}} - \tau_-$ the relations

$$H\Psi = \epsilon\Psi, \qquad H\Psi' = \epsilon'\Psi' \tag{6.4}$$

hold where

$$\Psi = (c^+\varphi^+ + c^-\varphi^-) \otimes \psi^0, \qquad \Psi' = (c^{-*}\varphi^+ - c^{+*}\varphi^-) \otimes \psi^0, \tag{6.5}$$

$$\psi^0 = \psi^0(\vec{r}), \tag{6.6}$$

Ψ is an actual state vector, and the quantum jump is caused by the crossing of levels ϵ and ϵ'.

Let at $t = t_{\text{jump}} + \tau_+$ the relations

$$H^\pm\Psi^\pm = \epsilon^\pm\Psi^\pm, \qquad H^\pm = H^\pm[\Psi^\pm], \tag{6.7}$$

hold, where

$$\Psi^\pm = \varphi^\pm \otimes \psi^\pm, \tag{6.8}$$

$$\psi^\pm(\vec{r}) = \psi^0(\vec{r} - \vec{R}^\pm), \tag{6.9}$$

and

$$R = |\vec{R}^+ - \vec{R}^-| \ll a_0. \tag{6.10}$$

Consider $H[\tilde{\Psi}]$ where

$$\tilde{\Psi} = c^+\Psi^+ + c^-\Psi^-. \tag{6.11}$$

In view of eq.(6.10), $V(\vec{r}; \tilde{\Psi})$ has only one minimum, so that $\tilde{\Psi}$ is not an eigenvector of $H[\tilde{\Psi}]$. Thus the quantum jump takes place with the result

$$\Psi^+ \quad \text{or} \quad \Psi^-, \qquad w^\pm = |c^\pm|^2. \tag{6.12}$$

To the vectors $\psi^\pm$ there correspond two different positions $\vec{R}^\pm$ of the pointer. Here no concept of decoherence is involved.

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